Errors in the Determination of Wind Speed by Doppler Radar

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ABSTRACT

A numerical model to simulate radar data is used for testing various estimators of the Doppler shift in Doppler radar echoes. Five estimators for the Doppler shift are considered: the pulse pair and poly-pulse pair algorithms in the correlation domain, least squares fitting to the power spectra in linear and logarithmic coordinates, and a matched filter in the spectral domain. An experiment with real data, to test the algorithms further and to assess the importance of small-scale wind fluctuations on radar performance, shows that geophysical limitations on the accuracy of the wind estimates are the dominant factor for observations with good signal-to-noise ratio.

1. Introduction

The use of Doppler radars such as VHF/UHF MST (Mesosphere-Stratosphere-Troposphere) radars, wind profilers (e.g., Rottger 1984) and Doppler weather radars (e.g., Doviak and Zrnić 1984) has been widespread for meteorological purposes over the last ten years. Statistical errors in the Doppler method of wind measurement have been considered on theoretical and numerical grounds for the case of weather radars (e.g., Sirmans and Bumgarner 1975; Zrnić 1979), in which short data records are as a rule used since the antenna is scanning. This discussion of the accuracy of various estimators of the Doppler shift makes particular reference to data having properties similar to those commonly found with VHF radar (wind profiler) observations. Some estimators of the Doppler shift of weather radar echoes, using both spectral and autocorrelation techniques, have been discussed by Sirmans and Bumgarner (1975), who concluded that for weather radars the autocorrelation method was superior. The algorithms to be examined are all unbiased by aliasing. The nomenclature introduced in Doviak and Zrnić (1984) is used here, where practical, with the addition of a "fading time," $\tau_{0.5}$, defined by the time for the autocorrelation function of a time series to fall to a value of 0.5 (Awe 1964b). The symbols used are defined in Table 1.

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2. The numerical model

Using a method of constructing artificial data with realistic statistical properties (May 1988), we drew a complex time series from a population of Gaussian distributed uncorrelated random numbers, in which the real and imaginary components are formed separately. This series was convolved with a Gaussian function so that the autocorrelation function of the time series had a Gaussian shape (Mitchell 1976, p. 115). The width of the Gaussian determines the correlation time of the signal which is inversely proportional to the spectral width (Table 1). The initial time series used must be longer than the "final" series to eliminate end effects. We use $\boldsymbol{S}$ to denote this complex series. A new time series $\boldsymbol{E}$ can be constructed from $\boldsymbol{S}$ so that it has a Doppler shift ω, given by

$$\boldsymbol{E} = \boldsymbol{S}e^{j\omega t+\phi} \quad (1)$$

where ϕ is an arbitrary phase constant. Noise can be added to the time series by simulating random numbers with a Gaussian distribution. The maximum signal-to-noise ratio (SNR) obtained with the simulations is about 30 dB because of such effects as digitization noise. Note that factors influencing the final shape of the observed spectrum such as window effects are automatically included. In all the simulations here the number of samples, M, was equal to 256 points (it has been shown in both the spectral and correlation domains [e.g., Zrnić 1977] that the accuracy of the estimators will be proportional to $1/\sqrt{M}$). The spectral width and SNR were varied.

3. Techniques for estimating the Doppler shift

a. Pulse pair technique (PP)

The Doppler shift of the returned signal is proportional to the slope of the phase of the autocorrelation

TABLE 1. List of symbols.

$S(f)$	power spectrum of the time series in units of frequency
$\rho(\tau)$	autocorrelation coefficient of the time series at a lag τ
σ_v	second central moment of power spectral peak in units of velocity $\left(v = \frac{\lambda}{2} f\right)$
σ_{vn}	second central moment of the power spectral peak normalized to the Nyquist interval $\left(\sigma_{vn} = \frac{2\sigma_v T_s}{\lambda}\right)$
SNR	signal-to-noise ratio
T_s	sampling interval for data taking
M	number of data points in the time series
$\tau_{0.5}$	lag such that the autocorrelation function falls to a value of 0.5 $\left(\tau_{0.5} \sim \frac{0.1874 T_s}{\sigma_{vn}}\right)$
λ	radar wavelength
n_f	nyquist frequency $\left(\frac{1}{2T_s}\right)$
arg( )	argument of a complex variable

function (at zero lag) of the returned signal. An estimator of the shift is the phase at the first lag divided by the value of the lag in time units (e.g., Woodman and Hagfors 1969; Woodman and Guillen 1974; Miller and Rochwarger 1972); that is,

$$v = \left(\frac{\lambda}{4\pi}\right) d\phi/dt \sim \left(\frac{\lambda}{4\pi}\right) \arg[\rho(T_s)]/T_s. \quad (2)$$

This is a special case of the autocorrelation estimator of the Dopper shift described in Doviak and Zrnić (1984) for data with uniformly spaced data points. A theoretical expression for the variance of this estimator has been obtained (Zrnić 1979; Doviak and Zrnić 1984):

$$\mathrm{var}(v) = \lambda^2 [32\pi^2 M \rho^2(T_s) T_s^2]^{-1} \left\{ \frac{[1 - \rho^2(T_s)] T_s}{2\sigma_{vn} T_s \sqrt{\pi}} + \frac{N^2}{S^2} + 2\left(\frac{N}{S}\right)[1 - \rho(2T_s)] \right\} \quad (3)$$

where the SNR (S/N) is measured after any coherent integration of the signals. The values of autocorrelation ρ are for the signal component only; therefore, the noise spike at zero lag in the autocorrelation function is interpolated through and ρ is renormalized. This expression was derived by using a perturbation analysis, which breaks down for cases with very narrow spectral widths or low values of SNR, unless the data records are very long (Doviak and Zrnić 1984).

Simulations were performed over a range of fading times that are considered to be typical for VHF radar records. Figure 1 shows rms fluctuations of the Doppler shift about the prescribed value $\langle (v - \bar{v})^2 \rangle^{1/2}$ for a variety of spectral widths and SNRs. Overall the agreement between theory and the results of these experiments is good. When the spectral width was very small the observed errors became very large. This is because data for very narrow spectral widths have very long fading times; thus the number of independent points ($\sim$ record length$/\tau_{0.5}$) becomes small. Also note that, as the SNR decreases, the theory underestimates the rms fluctuations of the measured Doppler shift at all spectrum widths, as was the case in the simulations of Zrnić (1977). The agreement between the theory, previous simulations, and our simulations give confidence in the method of generating data and to the subsequent testing of new techniques to analyze the simulated data.

b. Poly-pulse pair technique (PPP)

Some improvement in the estimates may be obtained by averaging more than one value of the phase divided by the lag (i.e., a weighted average of PP estimates from successive lags). The samples with lower values of correlation have larger error, so that the estimates are weighted. In these analyses the weighting function used is ρ^2 because the standard deviation of the correlation coefficient is proportional to $(1 - \rho^2)$ (Awe 1964a). If ρ^2 falls to less than 0.2 the sample is then given zero weight. The ρ^2 weighting of the autocorrelation phases was found experimentally to give better results than the ρ weighting. The PPP technique has been used previously (e.g., Strauch et al. 1978; Woodman 1985) and results in marked improvements for cases with poor SNR (Fig. 2) and small σ_{vn}, but no improvement at all was seen in the 30 dB case. The improvement comes about because of the averaging of the fluctuations in phase that are due to the contribution of the noise component of the signal. Woodman

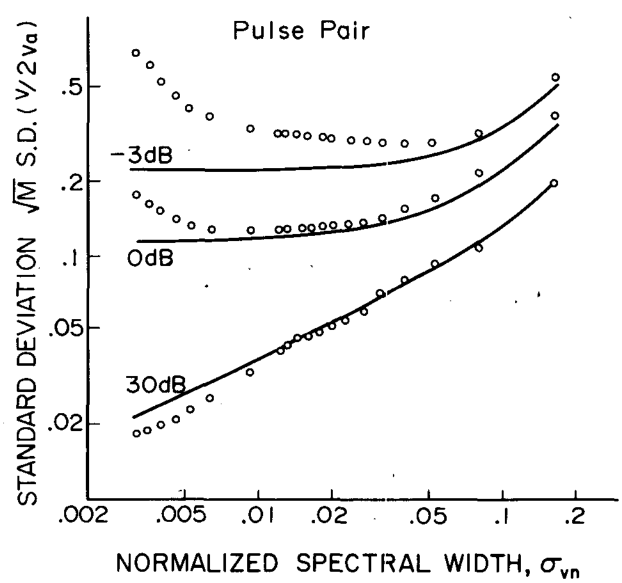

FIG. 1. Curves of the standard deviation of the pulse pair (PP) estimator against spectral width for various values of SNR. The number of data points in each time series is 256: *solid curves*, theoretical values; *circles*, the standard deviation of the estimate of the Doppler shift over 100 simulations for a given spectral width and SNR.

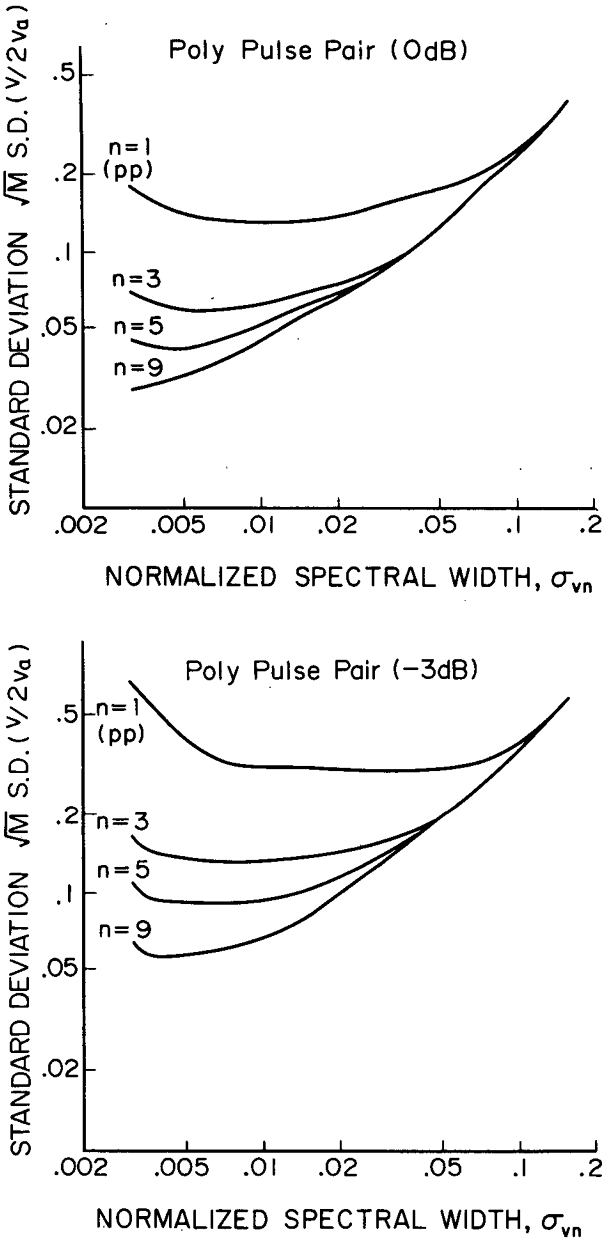

FIG. 2. The observed standard deviation of the PPP estimator (M = 256) vs spectral width for the averages of 1, 3, 5, and 9 values of phase. The simulation results for SNR values of (a) 0 dB, (b) −3 dB.

(1985) argued that the application of this method should give an improvement over the PP method for samples with good SNR, but the results of these simulations suggest that this does not occur. As discussed by May (1988), for autocorrelation functions with good SNR, the errors in adjacent estimates of the complex correlation are themselves highly correlated since they arise because of the finite number of independent samples rather than because of noise. Thus, averaging does not improve the estimates significantly for this case. One problem that arises is "aliasing" whereby the phase shift increases by the lag number and folds by factors of 2π at large lags. Aliasing is easy to account for the case of small errors and (or) small Doppler shifts (as in our case), but may be a significant problem for data with low SNR, leading to ambiguities in the determination of the Doppler shifts in real observations.

Four cases are considered in Fig. 2, where the numbers of autocorrelation coefficients used are 1 (PP technique), 3, 5, and 9. When the spectral width is wide the fading time of the autocorrelation function is short; the weight given to successive estimates decreases rapidly and the improvement by using large numbers of lags is small. The improvement also seems to occur at wider spectral widths when the data are noisier. As the number of coefficients to be included in the PPP is increased, the rate of improvement decreases, suggesting a limit as the values of autocorrelation measured decrease (Woodman 1985). For practical purposes, therefore, a limit in the number of coefficients used depends on typical values of $\tau_{0.5}$ as well as computing limitations. Although there was no significant improvement of the estimates in the case of high SNR, there was large improvements in the case of a SNR of about 0 dB. Sato and Woodman (1982) discussed a method of a least squares fitting procedure to the complex autocorrelation function, and the PPP method may be viewed as a simple approximation to the more complicated analysis they discussed.

It has been suggested that the use of maximum entropy methods (MEM) to find spectral peaks may offer some advantages (e.g., Klostermeyer 1986). Mahapatra and Zrnić (1983) also considered some simple forms of MEM suitable for real-time analysis. We examined these, but the PPP estimator with three correlation coefficients (requiring the same amount of calculations) gave superior results in every case. This contrasts with the results of Mahapatra and Zrnić (1983) who found that for short data records and good SNR the MEM gave smaller variances than the pulse pair method, although they also found that their MEM analysis was worse than the PPP for poor SNR.

c. Least squares fitting of the power spectrum—linear case

Doviak and Zrnić (1984) also considered the accuracy of the determination of the Doppler shift by a direct calculation of the first moment of the spectra (thresholded against noise). For the case of infinite SNR the errors are identical to errors in the PP method, but the spectral calculation is more strongly affected by noise (Doviak and Zrnić 1984). However, by use of such a spectral technique, it is often easier to edit data, for example, data contaminated by strong, fading ground clutter (e.g., Sato and Woodman 1982), to eliminate narrow spikes from the data (e.g., Hocking

1985) and to analyze bimodal spectra (Wakasugi et al. 1985). There is also the possibility of using a least squares fit (LSF) of an analytic function $F(a_i)$ (e.g., Gaussian or parabola) around the peak to obtain estimates of the Doppler shift. This is performed by minimizing a function $\chi^2(a_i)$ where $\chi^2(a_i)$ is defined by

$$\chi^2 = \sum_f [S(f) - F(a_i)]^2 \tag{4}$$

by varying the fitting parameters a_i. Before applying the least-squares fit, one views it possible to average adjacent spectral coefficients to decrease their variance, or take shorter datasets and average spectra. However, this does not improve the levels of accuracy of the fit compared with the raw single spectrum from the same total data record, because even though the reliability of the individual spectral estimates is increased, there is a corresponding decrease in the number of independent points in the spectra.

The least squares fitting routines applied to the spectra here use the method of Marquardt as described by Bevington (1969, p. 237). A choice must be made whether to fit the spectrum in linear coordinates or on a log scale. A Gaussian plus a constant is a good approximation to realistic data so this simple form is attractive but a severe drawback in use of a linear scale is that the variance of the individual spectral coefficients is the square of the magnitude of those coefficients. Thus when doing a least squares fit, the points with the largest values, where we want the fit to be best, also have the largest errors. This means that the χ^2 of the LSF is determined by only a few points around the peak, and that information from spectral estimates away from the peak (which have small amplitude) are not used (Yamamoto et al. 1988). Thus for the case of good SNR the errors are somewhat larger than in the PP method except for very narrow spectral widths where they are comparable (Fig. 3). A feature of the high SNR case is the almost linear dependence of the error on spectral width over the frequency range 0.01–0.04, in contrast to the $\sigma_v{}^2$ dependence found over the other frequencies (and in the PP method). For data with SNR of about 0 dB, the results are similar to the PPP when the number of correlation coefficients was about 5–9. The relative insensitivity to noise when the spectral width is narrow arises because the spectral peak of the signal still rises far above the noise level for such cases, unless the SNR is extremely bad (≤ -10 dB, depending on M) when catastrophic errors may occur because of errors in the initial guesses of the position of the spectral peaks.

d. Least squares fit to the logS(f)

The alternative, a logarithmic scale, has the immediate attraction that all the spectral coefficients have the same variance. In fact, Waldteufel (1976) constructed a maximum likelihood estimator that is similar

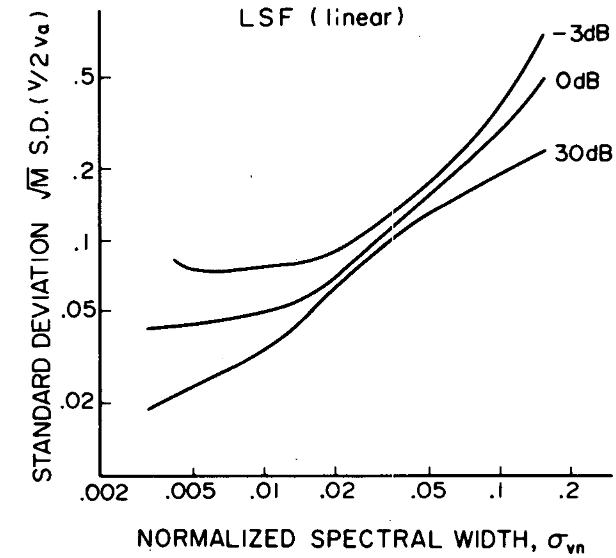

FIG. 3. The observed standard deviation for a least squares fit of the power spectrum in a linear domain against spectral width for SNR of 30 dB, 0 dB and −3 dB.

to a least squares fit in a logarithmic domain. Yamamoto et al. (1988) showed that for ideal Gaussian shaped spectra, the Cramer-Rao minimum variance bound, (CRB) (e.g., Doviak and Zrnić 1984, p. 113) may be approached with a LSF in the logarithmic domain for the ideal case of infinite SNR and a purely Gaussian spectrum (no window effects). The variance achieved with the CRB is an order of magnitude better than the PPP method. The main drawback here is that the choice of an analytic function to fit real data is not straightforward because of window effects, and the fitting is susceptible to spectral artifacts (e.g. Zrnić et al. 1977). The logarithm of a Gaussian function with an offset is not necessarily the best function since perturbations of the spectral shape by window effects may cause significant distortions in spectral shape for samples with good SNR, particularly if the spectral width is narrow (Sato and Woodman 1982). For our study we set $F(a_i)$ to an offset Gaussian function for the logarithmic case, as well as the linear case. That is a better approximation to the spectral shape for wide spectra compared with the log of a Gaussian, but it is still a poor choice for narrow spectra.

The results of this method indicate that for samples with good SNR, the least squares method gives better results than the PP for very wide spectra. However, the response of the variance is rather flat over narrower spectra, giving worse results over the range of spectral widths considered typical for MST radars (Fig. 4), probably because of the effects of window distortion making the choice of fitting function increasingly poor. A better choice of function would probably improve the performance of this estimator, but finding such a function is difficult since the importance of window

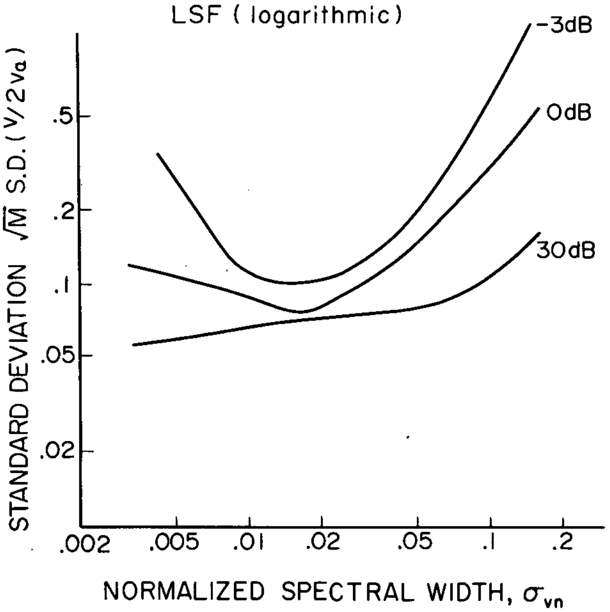

FIG. 4. The observed standard deviation as for Fig. 3, but for a least squares fit of the power spectrum in a logarithmic domain.

effects on the spectral shape is itself a function of spectral width (Sato and Woodman 1982). For data with poor SNR, the fit performs similarly to the PPP processor over most of the range of interest, but again worsens for narrow spectra. Also, fitting the spectrum on a log scale appears to be more sensitive to low SNR since the spectral peak does not rise as high over the noise level in this domain, compared with the linear case.

e. Matched filter analysis

Another technique is "Matched-filter Analysis" (Rottger 1986). In this analysis the power spectrum, $S(f)$ is calculated and then the circular convolution of $S(f)$,

$$m(\Delta f) = \sum_{-2f_N}^{2f_N} S(f-\Delta f)S(-f) \qquad (5)$$

is computed where the spectral coefficients outside of the Nyquist interval have been set to zero. This function has a maximum at exactly twice the mean frequency of the peak of $S(f)$ and is claimed to be less sensitive to asymmetrical and spiky spectra. If the padding of the spectra with zeros is not applied, the effective aliasing frequency is halved. It is possible to show that because the function $m(\Delta f)$ is the Fourier transform of the square of the complex conjugate of the autocorrelation function, and m can be calculated by successive FFTs (see Appendix). There are, however, some drawbacks to this technique. The first is that the problem of estimating the position of the matched filter peak remains. Thus the question of finding the optimum method is open. In fact one possible estimate for the first moment of m is half the argument of $\rho^2(T_s)$.

Figure 5 shows the results when a LSF in a linear domain is applied to the filter function. Even though m is much smoother than $S(f)$, the results for low SNR are similar to those obtained from a direct application of a least square fit to $S(f)$ (Fig. 3). With good SNR the Matched-filter method is as good or better than the PP technique. The Matched-filter method requires more computing, e.g., an additional two FFTs of double the length of the time series for each sample, offering little if any improvement. Therefore, it does not appear to be an attractive alternative except when "spiky" spectra occur.

4. Radar observations

We used real data in an experiment to study these various algorithms. For the case of a uniform wind field and for a radar beam pointing at an angle θ from the zenith, with a component of the horizontal wind in the plane of the radar beam (oriented in the east–west plane), U, and vertical wind, W, the line of sight Doppler shift u_1 will be:

$$u_1 = U\sin\theta + W\cos\theta. \qquad (6)$$

If in addition to this, we have a beam pointing in a direction $-\theta$, giving a line of sight velocity u_2 and a beam pointed to the zenith to give an estimate of W, denoted as w, we can then construct a quantity:

$$\delta u = u_1 + u_2 - 2w\cos\theta \qquad (7)$$

where δu should average to zero for a uniform wind field and the rms value of δu gives us a check on the

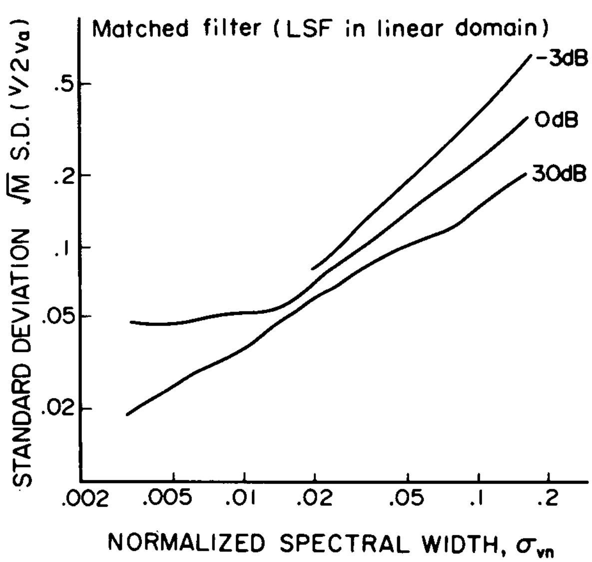

FIG. 5. The observed standard deviation as for Fig. 3, but for analysis of the matched filter function by least squares fitting.

errors in the wind measurement. Strauch et al. (1987) used a similar procedure to investigate the precision of hourly averages of the wind estimates. Also, δu can be converted into the uncertainty in the estimate of the horizontal velocity by dividing by $\sin\theta$ to give δU. Observations in the north–south plane are denoted by δv and δV. The theoretical error estimates may be used in the standard manner to examine how important the measurement errors are for the wind observations.

We obtained data for this experiment from the MU radar located at Shigaraki, Japan, operating at 46.5 MHz (see Fukao et al. 1985a,b for a comprehensive description of the radar). Four radar beams were pointed 10° from the zenith to the north, east, south and west and a fifth beam pointed to the zenith. The analysis techniques chosen were the PP, PPP, and least squares fitting in the linear and logarithmic domains. Time series of 128 points were collected in the range interval from 0 to 19.2 km, but useful results were obtained from only a minimum range of about 1 km because of the receiver recovery time. The range resolution was 150 m. The power spectra of five concurrent observations were averaged giving an effective time series length of 640 data points. The averaging of spectra is equivalent to taking a single long dataset and analyzing it assuming the spectral width is not too narrow. Thus we can apply the results of the simulations using the assumed dependence of the statistical errors on $\sqrt{M}$. The PP and PPP methods were applied by taking the Fourier transform of the averge power spectrum. The sampling interval was 0.07 seconds to give a Nyquist interval of $\sim \pm 22$ m s^{-1}.

Figure 6 shows an example of a scatter plot of individual estimates of δu and δv in a height regime with good SNR obtained using the PP technique. The correlation between the errors in the N–S plane and E–W plane occurs because the same vertical velocity corrections are used in the two measurements. The plots of the results with the other methods have a similar form; the standard deviation of δu and δv for four of the different analysis techniques are given in Table 2. The observations in the lower range of heights where the SNR is high, show that the errors incurred with the different methods were very similar; values of δu and δv were about 0.29 m s^{-1}. If we apply the theoretical results we should expect rms fluctuations of only about 0.13 m s^{-1} for typical spectral widths of about 0.7 m s^{-1}. Thus the observed fluctuations are much larger than expected. Note that the beams are horizontally separated by $\sim$1 km at a height of 3 km. Jasperson (1982) found that the rms wind variability was consistent with a $d^{1/3}$ law out to separations of about 20 km. Interpolating his results for a separation of 1 km gives an expected rms fluctuation in the horizontal wind velocity of 1 m s^{-1} (giving a δu value of $\sim$0.25 m s^{-1}). Jasperson's experiment was conducted over relatively flat terrain, and since the MU radar is located in a mountain range the variability may be greater.

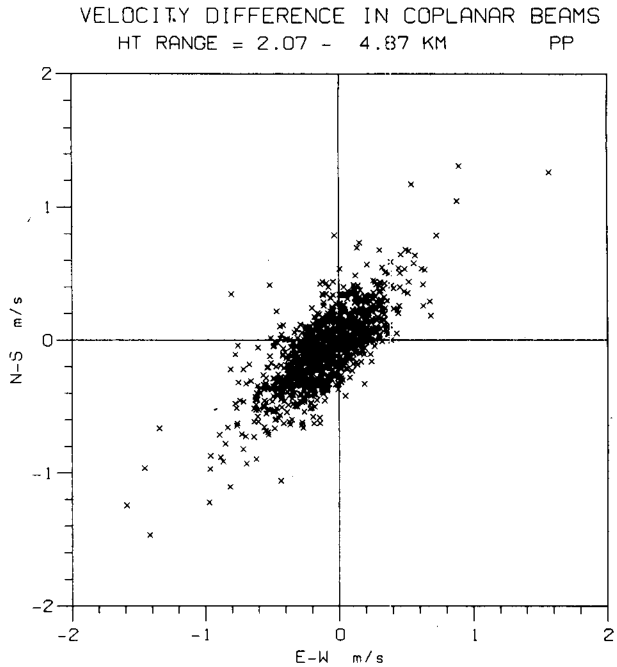

FIG. 6. Scatter plots of δu and δv from the MU radar experiment, with analysis by the PP method.

Differences in the vertical velocity may also contribute significantly, since $\cos\theta$ is almost unity for the MU radar measurements. Apparent large variations in the vertical wind velocity over scales of the order or the beam separations have been reported (Fukao et al. 1986). These wind variations are sufficient to reconcile the observed fluctuations with the theoretical expectations. When the LSF in logarithmic coordinates was used, a large difference was observed between δu and δv, which is not explained. The results in the upper height range where the SNR was close to 0 dB showed that the PP method was most susceptible to noise, as expected. In this case the dominant term contributing to δu and δv is the random statistical errors in the moment estimation, but the wind variability may also be a significant factor. As expected from the simulation results, the LSF in linear coordinates offered the best results, followed by the PPP when the SNR was poor. The PPP method applied here used only three values of phase. The use of more lags complicates the algorithm in order to avoid the effects of aliasing induced at the higher lags.

It should be noted that the contribution of random errors to δu is greater than that to actual wind measurements since the calculation of δu requires information from three beams, whereas the routine wind measurements require only two. For estimates of the horizontal wind (one component) with a vertical velocity correction, the random error will be a factor of $\sqrt{2}$ less than the contributions to δu, and the effect of spatial variability will also be less by a factor of about

TABLE 2. Values of δu and δv obtained with four different estimation techniques.

	PP		PPP-3		LSF (linear)		LSF (log)	
Range	2–5 km	7–10 km	2–5 km	7–10 km	2–5 km	7–10 km	2–5 km	7–10 km
δu (m s^{-1})	0.29	0.69	0.30	0.56	0.28	0.48	0.36	0.53
δv (m s^{-1})	0.30	0.70	0.30	0.56	0.27	0.49	0.29	0.57

0.8, assuming the $d^{1/3}$ law for spatial variability. An alternative is the adding of two estimates in one plane but opposite zenith angles (e.g., ~10°) and dividing by 2 sinθ. This removes the mean vertical contribution and reduces the statistical component by a factor of ($\sqrt{2}/4$) compared with the contributions to δu, but the effects of spatial variability remain. This has been considered in more detail by Koscielny and Doviak (1983). In many of the MST radars, significantly longer observation periods have been used by averaging several successive spectra. When the random errors are thus decreased, the limitation of the small-scale wind variability will be of the utmost importance.

Another feature to note is the nonzero mean of δu and δv in Fig. 6, which indicates mean differences in the wind vector between the three radar beams. This is probably due to mountain lee-waves which have been observed to dominate the vertical wind variability under some conditions (e.g., Ecklund et al. 1982; Nastrom et al. 1985). However, with only three beams in the plane there is insufficient information to study the lee waves in detail. This "mean" spatial variation of about 0.1 m s^{-1} over the beam separation implies uncertainties of about 0.25 m s^{-1} (mean difference 2 sinθ) in the estimates of the horizontal winds. Since, in the case of lee waves, this mean variation is slowly changing, the errors will be systematic over short time periods. In the case of highly convective conditions such as thunderstorms the effects may be even greater, although in this case they will also be transitory.

5. Conclusions

For data with good SNR, the PP estimator has the best performance; for noisy data, a least squares fit to the linear scale power spectrum is better. The PPP also offers much better performance than the PP at poor SNR and is equivalent when the SNR is high; thus the PPP may be a good compromise. A major limitation on the radar performance will be the small-scale variability of the wind so that this will mask marginal improvements in the analysis techniques when the SNR is good. Therefore, the geophysical limitation of the small-scale wind variability means that there is little gain in going to complicated algorithms (e.g., Sato and Woodman 1982; Waldteufel 1976) to obtain minimum variance estimates for many applications. Because the small-scale wind variability is of some interest, measurements of δu and δv may be useful as a measure of variability when corrected for the statistical errors in the Doppler shift estimates.

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APPENDIX

The Relation of the Matched Filter Function to the Autocorrelation Function

The matched filter analysis involves the construction of a filter function, the circular convolution of the power spectrum with itself:

$$m(F) = \int_{-f_N}^{f_N} S(f-F)S(-f)df \qquad \text{(A1)}$$

which has a maximum at twice the frequency of the maximum of $S(f)$. We wish to show that the Fourier transform of this is related simply to the autocorrelation function, which is the Fourier transform of $S(f)$. The Fourier transform of (A1), $M(\tau)$, is given by

$$M(\tau) = \int_{-\infty}^{\infty}\left[\int_{-f_N}^{f_N} S(f-F)S(-f)df\right]e^{-j2\pi\tau F}dF. \qquad \text{(A2)}$$

We now reverse the order of integration to obtain

$$M(\tau) = \int_{-f_N}^{f_N} S(-f)\int_{-\infty}^{\infty} S(f-F)e^{-j2\pi\tau F}dFdf. \qquad \text{(A3)}$$

Now consider the inner integral and make the substitution $F' = F - f(dF' = dF)$; thus the inner integral becomes

$$\int_{-\infty}^{\infty} S(-F')e^{-j2\pi\tau F'}dF'e^{-j2\pi\tau f} = \rho^*(\tau)e^{-j2\pi\tau f} \qquad \text{(A4)}$$

using the fact that the Fourier transform of $S(f)$ is $\rho(\tau)$, and thus the Fourier transform of $S(-f)$ is $\rho^*(\tau)$ because $S(f)$ is real (by definition). Substituting (A4) into (A3) then gives:

$$M(\tau) = \rho^*(\tau)\int_{-f_N}^{f_N} S(-f)e^{-j2\pi\tau f}df$$
$$= \rho^*(\tau)\rho^*(\tau). \qquad \text{(A5)}$$

Thus the matched filter function, $m(F)$ is a Fourier transform pair with the square of the complex conjugate of the autocorrelation function of the time series.

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