

CAUSE OF A MONOCHROMATIC INERTIA-GRAVITY WAVE BREAKING
OBSERVED BY THE MU RADAR

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Abstract. Mesospheric wind measurements made by the MU radar at Shigaraki (34.9°N, 136.1°E), Japan, on September 20, 1985, allow us to study gravity wave breaking. The wind data has shown that a monochromatic inertia-gravity wave was propagating vertically, indicating an exponential growth up to a mesospheric level with height. Above that level, however, the amplitude rather decreased. Moreover, the wave-form broke remarkably at a level. In this paper, we show that the breaking of the monochromatic wave can be connected with the convective instability in the wave field, estimating the local stability from the wind data.

Introduction

It is widely believed that the observed mesospheric circulation is maintained by the momentum deposition due to the breakdown of vertically propagating gravity waves [Lindzen, 1981; Holton, 1982; Matsuno, 1982]. The mechanisms that contribute significantly to the wave breaking are thought to be the convective and dynamical instabilities in the wave field [Fritts and Rastogi, 1985]. However, it has been hard to distinguish the breakings due to these instabilities, because of the inaccuracy in estimating the Richardson number and the static stability for the two conditions.

In our previous paper [Muraoka et al., 1987], we estimated almost all the parameters of a vertically propagating gravity wave observed with the MU radar. As a result we found that we had observed a stationary and monochromatic inertia-gravity wave. In such a particular case, the atmospheric stability in the wave field can be estimated with the aid of the dispersion and polarization relations for the wave. Therefore, it may be possible to identify the cause of a wave breaking observed in our radar experiment.

In this paper, we first present evidence for the breaking of a monochromatic gravity wave

observed by the MU radar. This is followed by an interpretation of the wave breaking based on the linear theory for monochromatic gravity waves.

Experimental Results

With the MU radar at Shigaraki (34.9°N, 136.1°E), Japan, we measured three components of mesospheric wind velocity on September 20, 1985. A meridional wind profile obtained during the period 1430-1635 LT is reproduced in Figure 1a. This figure clearly shows the existence of an almost monochromatic wave motion with a vertical wave length of 6 km in the background mean flow. Moreover, this wave was stationary (i.e., $c=0$) because the simultaneously observed echoing layers remained at the same heights for a long time. The characteristics of this wave have been reported in detail elsewhere [Muraoka et al., 1987; 1988]. The wave parameters are summarized in Table 1, together with the simultaneously obtained quantities of the mean state atmosphere. It is noted in Table 1 that the wave had a period of $5.6 h_2$ which means an inertia-gravity wave ($N_0^2 \gg \omega^2 \gg f^2$).

Figure 1a shows that the amplitude of the observed wave increased up to about 80 km with height. The amplitude increase is approximated well by $u_0(z_B) e^{(z-z_B)/2H}$ with $H=6.2$ km and $u_0=20.2$ ms⁻¹ at $z_B=81$ km, as indicated by chain lines. The value of u_0 is equal to the meridional mean flow speed at the same altitude. Above z_B , however, the amplitude rather decreases. It is further noted in Figure 1a that the wave-form breaks remarkably around 86 km. Thus it is clear that the gravity wave actually dissipated as it propagated upward into the mesosphere. In Figure 2, the change in the horizontal wind velocity with height is shown with a hodogram. While the clockwise elliptic polarization below 81 km with height indicates the vertical propagation of the gravity wave, it breaks down around 82 km.

Another example indicating the breaking of a monochromatic wave is shown in Figure 3a. The meridional wind profile was obtained during the period 1202-1406 LT on September 20, 1985. The wave parameters were almost the same as those in the case of Figure 1a [Muraoka et al., 1987]. It

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TABLE 1. Characteristics of gravity wave motion and mean atmosphere observed in the mesosphere on September 20, 1985.

Quantities	Notations	Values
WAVE FIELD		
Vertical wavelength	$\lambda_z = 2\pi/ m $	6 km
Horizontal wavelength	$\lambda_x = 2\pi/k$	400 km
Period	$2\pi/\omega$	5.6 h
Vertical phase velocity	ω/m	30 cms ⁻¹ downward
Horizontal phase velocity	ω/k	20 ms ⁻¹ southward
Ratio of the intrinsic to inertial frequency	f/ω	0.3
Doppler-shifted horizontal phase velocity	$c = \omega/k + \bar{u}$	0 ms ⁻¹
MEAN FIELD*		
Meridional wind velocity	\bar{u}	20 ms ⁻¹ northward
Vertical wind velocity	\bar{w}	20 cms ⁻¹ downward
Brunt-Väisälä frequency	N_0	0.021 s ⁻¹
Scale height	H_0	6.2 km
Temperature	T	210 K

* Evaluated around 75 km altitude.

is noted in Figure 3a that the wave breaks around 80 km remarkably. Below this height, the wave growth is approximated well by $u_0(z_B) e^{(z-z_B)/2H}$ with $H=6.2$ km and $u_0=17.3$ ms⁻¹ which equals the meridional mean flow speed at $z_B=79$ km. Note

that the wave amplitude rather decreases above 80 km with height.

Discussion

It has been suggested that the breaking of gravity waves arises from convective instability [Hodges, 1967; Lindzen, 1981]. The principal idea is that the wave produces a statically unstable state in the wave field, i.e., where

$$N^2 < 0. \quad (1)$$

Here N^2 is the total static stability defined as

$$N^2 \equiv (g/\theta)\theta_z \quad (2)$$

where θ is the total potential temperature, g is the acceleration of gravity, and the subscript denotes differentiation. The wave field is also known to be dynamically unstable where $Ri < 1/4$. Here Ri is the Richardson number defined as

$$Ri \equiv N^2 / (u_z^2 + v_z^2) \quad (3)$$

where u and v are the horizontal components of the wind velocity. The conditions under which gravity waves produce these unstable states in the wave field are reviewed by [Fritts and Rastogi, 1985].

For a monochromatic inertia-gravity wave ($N_0^2 \gg \omega^2 \gg f^2$), the relation between the horizontal wind and potential temperature perturbations is given by

$$\theta' = [i\bar{\theta}_z / m(c - \bar{u})]u' \quad (4)$$

where m is the vertical wavenumber, c is the horizontal phase velocity of the wave, overbars denote horizontal means, and primes denote perturbations. Substituting the differentiation of

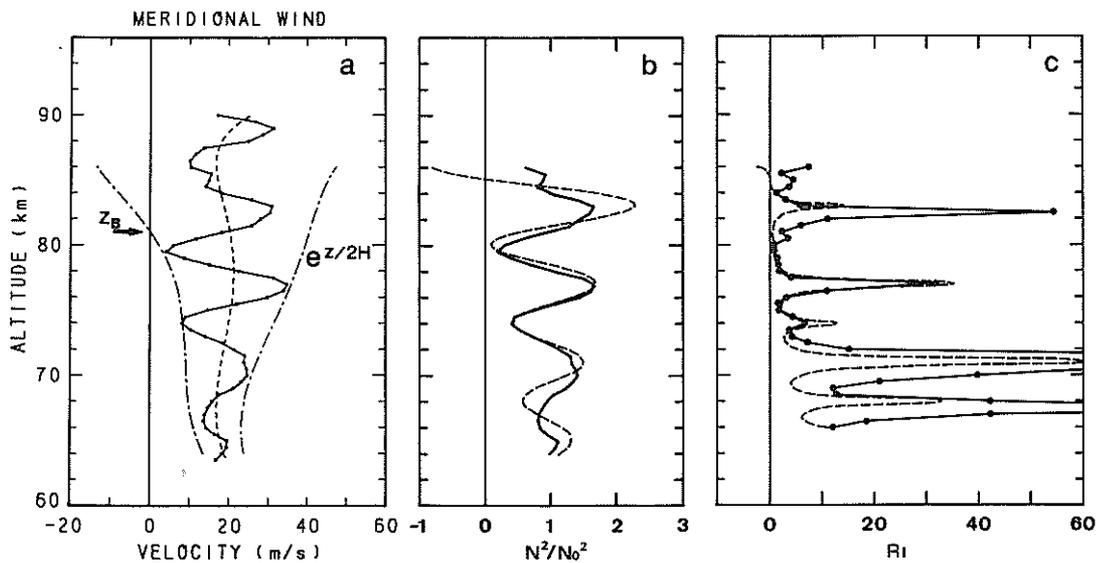


Fig. 1. A comparison between height profiles of (a) meridional wind velocity, (b) normalized total static stability (N^2/N_0^2) and (c) Richardson number (Ri) in the mesosphere. The solid, dashed and dot-dashed lines in the wind profile indicate a measured meridional wind, the mean flow and an exponential growth of the wind perturbation, respectively. The wind data came from the measurement during the period 1430-1635 LT on September 20, 1985. See text for the estimation of atmospheric stabilities.

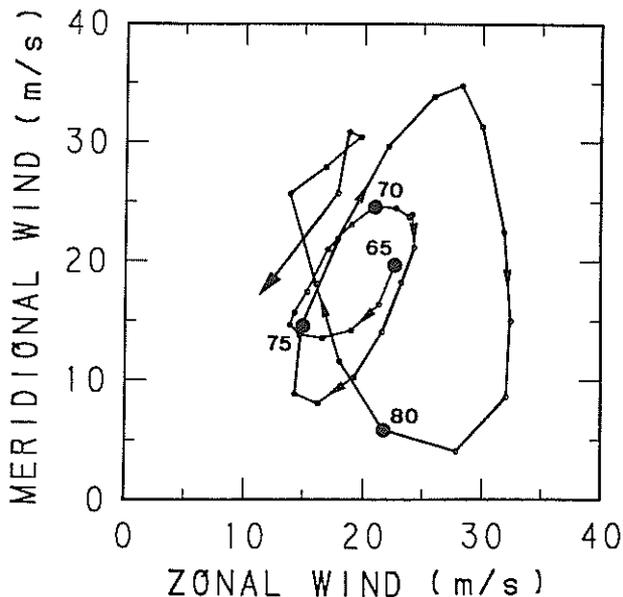


Fig. 2. A hodogram of the horizontal component of wind velocity measured during the period 1430-1635 LT on September 20, 1985. The altitudes at which the wind velocity was measured are indicated at every 5 km.

(4) with respect to z into (2) and considering that $\bar{\theta}_z$ is slowly varying with height, we obtain

$$N^2 = N_0^2 [1 - u' / (c - \bar{u})] \quad (5)$$

where $N_0^2 = (g/\bar{\theta})\bar{\theta}_z$ is the mean static stability. Thus, the condition (1) is equivalent to

$$u' / (c - \bar{u}) > 1 \quad (6)$$

for a monochromatic wave [Orlanski and Bryan, 1969]. With the use of (5), the Richardson number (3), as determined by mean and perturbation quantities, is given by

$$Ri = \frac{N_0^2 [1 - u' / (c - \bar{u})]}{(u_z'^2 + v_z'^2)} \quad (7)$$

Note that (5) and (7) can be further rewritten in terms of the wave amplitude u_0 and the phase function $\phi = k(x-ct) + mz$ if we write

$$\text{Re}[u'] = u_0 \cos \phi \quad (8)$$

as discussed by Fritts and Rastogi [1985].

From the measured wind data and the estimated wave parameters (cf. Table 1), we further estimated the local stability in the wave field, N^2 . In Figures 1b and 1c, the height changes of N^2/N_0^2 and Ri are compared with the wave motion in Figure 1a. The solid lines in Figures 1b and 1c indicate the N^2/N_0^2 and Ri profiles estimated from the measured meridional mean and perturbation wind velocity in (5) and (7), respectively. The dashed lines indicate the profiles estimated from the wave parameters. In this estimation, we assumed the exponential growth of the amplitude ($u_0 = 20.2 \text{ ms}^{-1}$ at 81 km) above z_B and $\phi = 0$ at 74 km. The solid and dashed lines in the N^2/N_0^2 and Ri profiles agree well with each other below 80 km where the observed wave is growing exponentially with height, as is expected. A phase difference between the two N^2/N_0^2 profiles is seen below 70 km. This indicates that the vertical wavelength was slowly varying with a change in the mean state atmosphere, as suggested from the dispersion relation

$$|m| = N_0 / |c - \bar{u}| \quad (9)$$

for inertia-gravity waves.

We see in Figures 1b and 1c that the wave field becomes convectively unstable ($N^2 < 0$) around 86 km, assuming the further exponential growth of the vertically propagating wave. In relation to this instability, the wave-form has broken around there in reality (Figure 1a). Figure 1b further shows that the atmosphere is returning to its original state (i.e., $N^2 \rightarrow N_0^2$) around the breaking

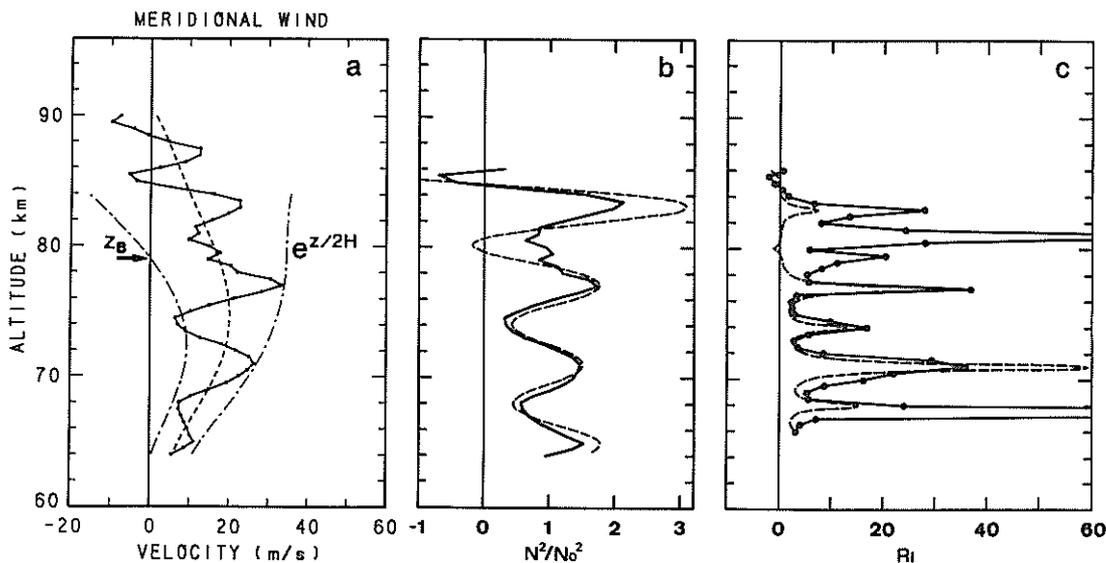


Fig. 3. Same as Figure 1 except for observation between 1202 and 1406 LT on September 20, 1985.

level. We have shown in Figure 1a that the wave starts dissipating at a height indicated by z_B , where the amplitude becomes equal to the mean flow. In spite of the sufficient amplitude to satisfy (6), the wave field did not produce convectively unstable state around there because of the phase shift (Figure 1b). Since the local Richardson number in the exponentially growing wave field becomes very small (~ 0.5) around z_B (Figure 1c), there is a possibility that the dissipation arises from the dynamically unstable state.

Figure 3 is another comparison between the observed gravity wave motion and the height changes of the static and dynamic stabilities in the wave field. The height changes of N^2/N_0^2 and Ri were estimated from the measured wind velocity and the estimated wave parameters (Table 1), using the same method as described above for Figure 1. Also in this case, the agreement between the two profiles is good below 79 km where the wave is growing exponentially with height. Assuming the further exponential growth ($u_0=17.3 \text{ ms}^{-1}$ at 79 km) and $\phi=0$ at 74 km, we see in Figures 3b and 3c that the wave field becomes convectively unstable ($N^2 < 0$) around 80 km, where the wave clearly breaks down. Figures 3b and 3c also show that the atmosphere is returning to its original state (i.e., $N^2 \rightarrow N_0^2$ and $Ri \gg 1$) around the breaking level.

Above the wave breaking level, the eddy diffusion needed to prevent the further wave growth is determined from the linear saturation theory as [Lindzen, 1981],

$$D = \frac{1}{m^2} \left| \frac{\omega}{m} \right| \left(\frac{1}{2H} - \frac{3}{2} \frac{\bar{u}}{\bar{u}-c} \right). \quad (10)$$

Using our observational results and assuming that m and H are slowly varying with height, we estimate the diffusion coefficient at the breaking level. Since $\bar{u} = -16.7 \text{ ms}^{-1}$ and $\bar{u} = 1.13 \times 10^4 \text{ s}^{-1}$ at 86 km in the case shown in Figure 1, (10) yields $D \sim 25 \text{ m}^2 \text{ s}^{-1}$ there. In the case shown in Figure 3, we obtain $D \sim 56 \text{ m}^2 \text{ s}^{-1}$ at 80 km since $\bar{u} = -16.1 \text{ ms}^{-1}$ and $\bar{u} = 1.35 \times 10^4 \text{ s}^{-1}$ there. The eddy diffusion coefficients are somewhat smaller than the values suggested by Lindzen [1981] and Holton [1982]. Taking into account the remarkable wave damping around the breaking level, we may expect that the resulting diffusion becomes much greater than that estimated from the linear saturation theory.

Concluding Remarks

In this paper, we have shown that a monochromatic inertia-gravity wave broke down as it propagated vertically. The wave breaking was seen at the level where the wave field itself would become convectively unstable if the wave continued to propagate upwards with the exponential growth rate. This result suggests that the breaking of the monochromatic wave arises from the convective instability in the wave field. The remarkable wave damping around the breaking level strongly suggests the generation of turbulence, which leads to a large eddy diffusion comparable to the earlier estimates by Lindzen [1981] and Holton [1982]. This result will give a better observational basis to the parameterization for

the eddy diffusion generated by the wave breaking, although the event discussed in this paper was the only example indicating the explicit wave breaking in our several experiments with the MU radar. The difficulty in detecting such breaking events is, in most cases, caused by the change in mesospheric VHF echo power depending on the rather complicated scattering process [Röttger, 1987]. However, it goes without saying that further observational effort will clarify the wave breaking mechanism.

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